Fuglede's spectral set conjecture on cyclic groups
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Fuglede's conjecture (1974) states that a bounded measurable subset in R^d accepts an orthogonal basis of exponential functions (i.e. it is spectral) if and only if it tiles the space with a discrete set of translations. This conjecture turned out to be false by Tao's counterexample in 2003. Using Tao's ideas, counterexamples in finite Abelian groups such as Z_N^d can be lifted to counterexamples in R^d, thus shifting the interest on this conjecture to this setting in recent years. This has been successful for d>2, but the conjecture is still open for d=1,2.

Some recent results in the cyclic group setting will be presented in this talk, which are connected to the work of Coven-Meyerowitz and Laba on tiling subsets of Z, as well as the structure of vanishing sums of roots of unity. This is partially joint work with Mihalis Kolountzakis.